On a recently proposed metric linear extension of general relativity to explain the Pioneer anomaly

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Abstract

Recently, Jaekel and Reynaud put forth a metric linear extension of general relativity which, in the intentions of its proponents, would be able, among other things, to provide a gravitational mechanism for explaining the Pioneer anomaly without contradicting either the equivalence principle or what we know about the planetary motions. In this paper we perform an independent test of such an hypothesis by showing that the planets' orbits are, in fact, affected by the suggested mechanism as well, and comparing the resulting effects with the latest observational determinations. It turns out that the predicted perihelion precessions, expressed in terms of an adjustable free parameter $\zeta_{\rm P}M$ set equal to the value used to reproduce the magnitude of the Pioneer anomalous acceleration, are quite different from the observationally determined extra-advances of such Keplerian element for the inner planets. Conversely, the values obtained for $\zeta_{\rm P}M$ from the determined perihelion extra-rates of the inner planets turn out to be in disagreement with the value which would be required to accommodate the Pioneer anomaly. As a consequence, the suggested explanation for the Pioneer anomaly, based on the assumption that $\zeta_{\rm P}$ is constant throughout the Solar System, should be rejected, at least in its present form.

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1 Introduction

In order to find an explanation of gravitational origin for the anomalous acceleration of about $(8.74 \pm 1.33) \times 10^{-10}$ m s^{-2} experienced by the Pionner 10/11 spacecraft after they passed the threshold of 20 AU (Anderson et al.

1998; 2002), Jaekel and Reynaud (2005a; 2005b) proposed to use a suitable metric linear extension of General Relativity with two potentials $\Phi_{\rm N}$ and $\Phi_{\rm P}$. In the gauge convention of the PPN formalism its space-time line element, written in isotropic spherical coordinates, is (Jaekel and Reynaud 2005b)

$$ds^2 = g_{00}c^2dt^2 + g_{rr}[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)], \tag{1}$$

with

$$\left\{\begin{array}{l} g_{00} = 1 + 2\Phi_{\rm N}, \\ \\ g_{rr} = -1 + 2\Phi_{\rm N} - 2\Phi_{\rm P}. \end{array}\right. \tag{2}$$

In order to accommodate the Pioneer anomaly the following simple model

$$\Phi_j(r) = -\frac{G_j M}{c^2 r} + \frac{\zeta_j M r}{c^2}, j = {\rm N}, {\rm P}, \tag{3}$$

has been used (Jaekel and Reynaud 2005a; 2005b). It is determined by four constants: the Newtonian constant $G_{\rm N}$ and the three small parameters $G_{\rm P}, \zeta_{\rm N}$ and $\zeta_{\rm P}$ which measure the deviation from general relativity. In the intentions of Jaekel and Reynaud, their theory should be able to explain the occurrence of the Pioneer anomaly a) without violating either the existing constraints from the planetary motions b) or the equivalence principle. The latter goal is ensured by the metric character of the proposed extension of general relativity. In regard to a), they first focus their attention to the modification of the Newtonian potential. By using the orbits of Mars and the Earth they get an upper bound $|\zeta_{\rm N} M| \simeq 5 \times 10^{-13}$ m s^{-2} (Jaekel and Reynaud 2005b) which excludes that $\zeta_{\rm N} Mr/c^2$ is capable to account for the anomalous Pioneer acceleration. The key point of their line of reasoning in explaining the Pioneer anomaly without contradicting our knowledge of the planetary orbits consists in considering from the simple expression of eq. (3) for $\Phi_{\rm P}$ the following extra-kinetic radial acceleration[1]

$$A_{\rm P} = 2\zeta_{\rm P} M \frac{v_r^2}{c^2}, \tag{4}$$

where v_r is the radial component of the velocity of the moving body, in identifying it with the source of the Pioneer anomalous acceleration by getting[2] $\zeta_{\rm P} M = 0.25$ m s^{-2} and in claiming that eq. (4) cannot affect the

[1]The contribution of $G_{\rm P}$ is found to be negligible.

[2]The almost constant value $v_r = 1.2 \times 10^4$ m s^{-1} has been used for both the Pioneer spacecraft.

planetary motions because almost circular. Conscious of the fact that independent tests are required to support their hypothesis and since no accurate and reliable data from other spacecraft are available to this aim, Jaekel and Reynaud (2005b) propose to perform light deflection measurements because $\Phi_{\rm P}$ affects the motion of electromagnetic waves as well. A re-analysis of the Cassini (Bertotti et al. 2003) data is suggested (Jaekel and Reynaud 2005b).

In this paper we will show that, in fact, the extra-kinetic acceleration of eq. (4) does also affect the orbital motions of the planets in such a way that it is possible to compare the resulting features of motion with the latest data from planetary ephemerides, thus performing right now a clean and independent test of the hypothesis that eq. (4) is able to accommodate the Pioneer anomaly. We will also discuss the feasibility of the proposed light deflection measurements in view of the results obtained from the perihelia test.

2 The orbital effects of the kinetic acceleration and comparison with the latest data

The Russian astronomer E.V. Pitjeva has recently processed almost one century of data of all types in the effort of continuously improving the EPM2004 planetary ephemerides. Among other things, she also determined residual advances of the perihelia ω of the inner (Pitjeva 2005) and outer (Pitjeva 2006) Solar System planets as fit-for parameters of a global solution in which she contrasted, in a least-square way, the observations to their predicted values computed with a complete set of dynamical force models including all the known Newtonian and Einsteinian features of motion. As a consequence, any unmodelled force, as it would be the case for a Pioneer-like one if present in Nature, is entirely accounted for by the so-obtained residual perihelia advances.

In order to make a direct comparison with them, we will now analytically work out the secular, i.e. averaged over one orbital revolution, effects induced by the extra-kinetic acceleration of eq. (4) on the pericentre of a test particle. To this aim, we will treat eq. (4) as a small perturbation of the Newtonian monopole. In order to justify this assumption, we will first evaluate the average of eq. (4) and, then, we will compare it with the the Newtonian mean accelerations throughout the Solar System. To this aim, we must evaluate eq. (4) onto an unperturbed Keplerian ellipse by means of

$$v_r = \frac{nae\sin f}{\sqrt{1-e^2}}, \tag{5}$$

where a is the semimajor axis, e is the eccentricity, $n = \sqrt{GM/a^3}$ is the (unperturbed) Keplerian mean motion and f is the true anomaly. Subsequently, the average over one orbital period $P = 2\pi/n$ has to be performed. It is useful to adopt the eccentric anomaly E by means of the relations

$$\left\{\begin{array}{l} dt = \frac{(1-e\cos E)}{n}dE, \\ \\ \cos f = \frac{\cos E-e}{1-e\cos E}, \\ \\ \sin f = \frac{\sin E\sqrt{1-e^2}}{1-e\cos E}. \end{array}\right. \tag{6}$$

By using

$$\int_0^{2\pi}\frac{\sin^2 E}{1-e\cos E}dE = \frac{2\pi}{e^2}\left(1-\sqrt{1-e^2}\right), \tag{7}$$

we get

$$\langle A_{\mathrm{P}}\rangle = \frac{2\zeta_{\mathrm{P}}Mn^2a^2}{c^2}\left(1-\sqrt{1-e^2}\right). \tag{8}$$

Eq. (8) can, now, be compared with

$$\langle A_{\mathrm{N}}\rangle = \frac{GM}{a^2\sqrt{1-e^2}}, \tag{9}$$

The results are in Table 1 From it it clearly turns out that the use of the perturbative scheme is quite adequate for our purposes. The Gauss equation for the variation of ω under the action of an entirely radial perturbing acceleration A_r is

$$\frac{d\omega}{dt} = -\frac{\sqrt{1-e^2}}{nae}A_r\cos f. \tag{10}$$

After being evaluated onto the unperturbed Keplerian ellipse by using eq. (5), eq. (4) must be inserted into eq. (10); then, the average over one orbital period has to be taken. By means of

$$\int_0^{2\pi}\frac{\sin^2 E(\cos E-e)}{(1-e\cos E)^2}dE = \frac{2\pi}{e^3}\left(-2+e^2+2\sqrt{1-e^2}\right), \tag{11}$$

it is possible to obtain

$$\frac{d\omega}{dt} = -\frac{2\zeta_{\mathrm{P}}Mna\sqrt{1-e^2}}{c^2e^2}\left(-2+e^2+2\sqrt{1-e^2}\right). \tag{12}$$

Table 1: Average Pioneer and Newtonian accelerations for the Solar System planets, in m s^{-2}. For $\langle A_{\rm P}\rangle$ the expression of eq. (8) has been used with $\zeta_{\rm P}M = 0.25$ m s^{-2}.

Planet	$\langle A_{\rm P}\rangle$	$\langle A_{\rm N}\rangle$
Mercury	2×10^{-10}	4×10^{-2}
Venus	1×10^{-13}	1×10^{-2}
Earth	6×10^{-13}	6×10^{-3}
Mars	1×10^{-11}	2×10^{-3}
Jupiter	1×10^{-12}	2×10^{-4}
Saturn	8×10^{-13}	6×10^{-5}
Uranus	2×10^{-13}	1×10^{-5}
Neptune	6×10^{-15}	6×10^{-6}
Pluto	4×10^{-12}	4×10^{-6}

Note that eq. (12) is an exact result. It may be interesting to note that the rates for the semimajor axis and the eccentricity turn out to be zero; it is not so for the mean anomaly $\mathcal{M}$, but no observational determinations exist for its extra-rate. We will now use eq. (12) and $\zeta_{\rm P}M = 0.25$ m s^{-2}, which has been derived from eq. (4) by imposing that it is the source of the anomalous Pioneer acceleration, to calculate the perihelion rates of the inner[3] planets of the Solar System for which estimates of their extra-advances accurate enough for our purposes exist (Pitjeva 2005). The results are summarized in Table 2

It clearly turns out that the determined extra-advances of perihelia are quite different from the values predicted in the hypothesis that eq. (4) can explain the Pioneer anomaly. In Table 3 we show the values of $\zeta_{\rm P}M$ which can be obtained from the determined extra-advances of perihelia (Pitjeva 2005); as can be noted, all of them are far from the value which would be required to obtain the correct magnitude of the anomalous Pioneer acceleration. The experimental intervals obtained from Mercury, the Earth and Mars are compatible each other; Venus, instead, yields values not in agreement with them. This fact can be explained by noting that its perihelion is a bad observable due to its low eccentricity ($e_{\rm Venus} = 0.00677$). By applying the Chauvenet criterion we reject the value obtained from the Venus perihelion since it lies at almost 2σ from the mean value of the distribution

[3] The extra-perihelion rates of the outer planets (Jupiter, Saturn, Uranus) have recently been determined in a very preliminary way (Pitjeva 2006); it turns out that the realistic uncertainties are still so large that they cannot be used for a meaningful comparison.

Table 2: (P): predicted extra-precessions of the longitudes of perihelia of the inner planets, in arcseconds per century, by using eq. (12) and $\zeta_{\rm P}M = 0.25$ m s^{-2}. (D): determined extra-precessions of the longitudes of perihelia of the inner planets, in arcseconds per century. Data taken from Table 3 of (Pitjeva 2005). It is important to note that the quoted uncertainties are not the mere formal, statistical errors but are realistic in the sense that they were obtained from comparison of many different solutions with different sets of parameters and observations (Pitjeva, private communication 2005).

	Mercury	Venus	Earth	Mars
(P)	1.8323	0.001	0.0075	0.1906
(D)	-0.0036 ± 0.0050	0.53 ± 0.30	-0.0002 ± 0.0004	0.0001 ± 0.0005

Table 3: Values of $\zeta_{\rm P}M$, in m s^{-2}, obtained from the determined extra-advances of perihelia (Pitjeva 2005). After discarding the value for Venus, the weighted mean for the other planets yields $\zeta_{\rm P}M = -0.0001 \pm 0.0004$ m s^{-2}. The Pioneer anomalous acceleration is, instead, reproduced for $\zeta_{\rm P}M = 0.25$ m s^{-2}.

	Mercury	Venus	Earth	Mars
$\zeta_{\rm P}M$	-0.0005 ± 0.0007	91 ± 51	-0.006 ± 0.013	0.0001 ± 0.0006

of Table 3. The weighted mean for Mercury, the Earth and Mars is, thus, $\langle \zeta_{\rm P} M \rangle_{\rm w} = -0.0001$ m s^{-2} with a variance, obtained from $1/\sigma^2 = \sum_i (1/\sigma_i^2)$, of 0.0004 m s^{-2}.

An analysis involving the perihelia of Mars only can be found in (Jaekel and Reynaud 2006). In it Jaekel and Reynaud present a nonlinear generalization of their model, and an explicit approximate expression of the perihelion rate different from eq. (12) can be found; it[4] is calculated with $\zeta_{\rm P} M = 0.25$ m s^{-2} yielding a value for the Martian perihelion advance which is about one half of our value in Table 2. Even in this case, the results by Pitjeva (2005) for Mars would rule out the hypothesis that the Pioneer anomaly can be explained by the proposed nonlinear model. By the way, in (Jaekel and Reynaud 2006) no explicit comparison with published or publicly available data is present.

All the previous considerations are based on the simple model of eq. (3), with $\zeta_{\rm P}$ constant over the whole range of distances from the radius of the Sun to the size of the Solar System. Jaekel and Reynaud (2005a; 2005b; 2006), in fact, leave generically open the possibility that, instead, $\zeta_{\rm P}$ may vary with distance across the Solar System, but neither specific empirical or theoretical justifications for such a behavior are given nor any explicit functional dependence for $\zeta_{\rm P}(r)$ is introduced.

3 The deflection of light

The results for $\zeta_{\rm P} M$ from the determined extra-rates of the perihelia of the inner planets allow us to safely examine the light deflection measurements originally proposed by Jaekel and Reynaud as independent tests of their theory; indeed, the values of Table 3 certainly apply to the light grazing the Sun, also in the case of an hypothetical variation of $\zeta_{\rm P}(r)$ with distance. In (Jaekel and Reynaud 2005a) they found the following approximate expression for the deflection angle induced by $\zeta_{\rm P}$

$$\psi_{\rm P} = -\frac{2\zeta_{\rm P} M \rho}{c^2} L, \tag{13}$$

where ρ is the impact parameter and L is a factor of order of unity which depends logarithmically on ρ and on the distances of the emitter and receiver to the Sun. For $\rho = R_\odot$, $L \sim 1$, and $\zeta_{\rm P} M = -0.0001$ m s^{-2}, eq.

[4] More precisely, in (Jaekel and Reynaud 2006) an explicit expression for the adimensional perihelion shift after one orbital period, in units of 2π, i.e. $(\dot\omega P)/2\pi$, can be found; a direct comparison with our results can be done simply by multiplying their formula by n and making the conversion from s^{-1} to arcseconds per century.

(13) yields a deflection of only -0.3 microarcseconds, which can be translated into an equivalent accuracy of about 2×10^{-7} in measuring the PPN parameter γ with the well-known first-order Einsteinian effect (1.75 arcseconds at the Sun's limb). Such a small value is beyond the presently available possibilities; indeed, the Cassini test (Bertotti et al. 2003) reached a 10^{-5} level, which has recently been questioned by Kopeikin et al. (2006) who suggest a more realistic 10^{-4} error. Instead, it falls within the expected 0.02 microarcseconds accuracy of the proposed LATOR mission (Turyshev et al. 2006), which might be ready for launch in 2014. Also ASTROD (Ni 2002) and, perhaps, GAIA (Vecchiato et al. 2003), could reach the required sensitivity to measure such an effect. However, because of technological and programmatic difficulties, the launch of an ASTROD-like mission is not expected before 2025. GAIA is scheduled to be launched in 2011 (http://gaia.esa.int/science-e/www/area/index.cfm?fareaid=26).

4 Conclusions

In this paper we have performed an independent test of the hypothesis that the Pioneer anomaly can be explained by a particular form of a recently proposed metric linear extension of general relativity (Jaekel and Reynaud 2005a; 2005b) without contradicting either the equivalence principle or the constraints from planetary motions. Such a mechanism is based on a simple explicit model involving, among other things, the occurrence of an additional potential in the metric coefficient g_{rr} parameterized in terms of an adjustable free constant $\zeta_{\rm P}$ and linearly varying with the distance r.

We have first compared the effects that the resulting extra-acceleration, expressed in terms of the parameter $\zeta_{\rm P}M$ set to the value which yields the magnitude of the Pioneer anomalous acceleration, would also have on the Solar System planetary motions with the latest available observational determinations (Pitjeva 2005). It turns out that the determined perihelion rates of the inner planets rule out the proposed gravitational mechanism for explaining the Pioneer extra-acceleration, at least in its present form. Conversely, the determined extra-rates of the perihelia of the inner planets have been used to measure $\zeta_{\rm P}M$ independently of the Pioneer effect: it turns out that the so obtained value for such a constant is three orders of magnitude smaller than it would be required to reproduce the anomalous Pioneer acceleration. Another independent test of the proposed model is represented, in principle, by light deflection measurements in the proximity of the Sun because $\zeta_{\rm P}$ also affects the propagation of the electromagnetic waves. It turns out that the deflection angle resulting from the value of $\zeta_{\rm P} M$ obtained with the perihelia extra-rates would be too small to be detected with the present-day technology; future missions like LATOR, ASTROD and GAIA will, instead, be able to measure such an effect.

The previous conclusions are based on the assumption that $\zeta_{\rm P}$ is constant throughout the Solar System; Jaekel and Reynaud, in fact, admit the possibility that it may vary with the distance, but without further details.

The results presented here further enforces the conclusions of other studies (Iorio and Giudice 2006; Tangen 2006) pointing towards an exclusion of a gravitational origin of the anomalous features of motion experienced by the Pioneer spacecraft.

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